The \$1,000,000 Math Mystery: The Riemann Hypothesis

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A Million-Dollar Question

The Clay Mathematics Institute is offering \$1,000,000 to anyone who can solve the Riemann Hypothesis. But why is it so hard?

Look down and you will see...

Wait, what is this??!!

The function $\zeta(s)$ has zeros at the negative even integers $-2, -4, \ldots$ and one refers to them as the *trivial zeros*. The other zeros are the complex numbers $\frac{1}{2} + i\alpha$ where α is a zero of $\xi(t)$. Thus, in terms of the function $\zeta(s)$, we can state

The nontrivial zeros of $\zeta(s)$ have real part equal to $\frac{1}{2}$.

What is the Riemann Zeta Function?

Prime numbers (2, 3, 5, 7, 11...) don't follow an obvious pattern. Mathematicians have been trying to figure out a formula that predicts where the next prime will appear. The Riemann Hypothesis suggests that there is a hidden structure behind their distribution!

The Riemann Zeta Function is written as:
$$\zeta(s)=1+rac{1}{2^s}+rac{1}{3^s}+rac{1}{4^s}+rac{1}{5^s}+\dots$$

It is a sum that involves all positive numbers raised to the power of s. When you plug in different numbers for s, you get interesting patterns related to prime numbers. (Don't ask me why did he thought of this, because I am not Riemann.)

But here's the catch: the function has zeros (points where it equals 0), and those zeros seem to follow a strange rule.

What is the hypothesis itself?

Bernhard Riemann (a German mathematician) guessed that all the "non-trivial" zeros of the function lie on a straight vertical line in the complex number plane at Re(s) = 1/2.

$$s=rac{1}{2}+bi$$

Where b is any real number. This means that every time the zeta function equals zero (excluding some obvious cases), it happens at a point where the real part of s is exactly 1/2. If this is true for ALL zeros, then the Riemann Hypothesis is correct. But no one has been able to prove it yet!

Why does it matter if it is solved?

If the Riemann Hypothesis is true, it means prime numbers aren't random—they follow a deep pattern! This would have huge effects on:

- Cryptography (keeping digital information safe)
- Quantum physics
- Computer science
- Number theory (the study of numbers

But if it's false, it means our entire understanding of prime numbers could be wrong!

Why is it hard to prove even we have computers?

- 1. It deals with complex numbers It's not just regular numbers; it involves numbers like 3+4i
- 2. It involves infinity Mathematicians have checked trillions of cases, but proving it for all numbers is the problem.
- 3. We might need new math Some believe that solving RH requires math that hasn't even been discovered yet!

Are you the next math genius? Check it out!

The Riemann Hypothesis is considered the greatest unsolved problem in pure mathematics. If someone proves it, they win \$1,000,000 and change math forever!

If you really solved it, you'd become a millionaire, a math legend, and the person who cracked a 150-year-old mystery!

You can see the whole introduction & problem here: https://www.claymath.org/wp-content/uploads/2022/05/riemann.pdf

The end

Even today, some math problems remain unsolved. Maybe one day, YOU could be the one to solve them!

